An image reconstruction framework for polychromatic interferometry

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Introduction

- **Context**,
  - there is a need of dedicated image reconstruction algorithms for polychromatic interferometry,
  - spatio-spectral correlations put strong constraints for image reconstruction (SNIFS (Bongard et al. 2011), MUSE (Soulez et al. 2013)),
  - one of the goal of the POLCA project,
- **Outlines**
  - a new architecture for the polychromatic "MiRA 3D" algorithm,
  - example in the GRAVITY case.

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Image reconstruction

- **goal:** *estimate 3D intensity distribution* $I(\boldsymbol{\theta}_n, \lambda_\ell)$ *of the observed object*
- **we have** (measurements):
  - interferometric measurements (visibilities, closure. . .) $\boldsymbol{m}$,
  - photometric measurements s,
- **we know** (priors):
  - positivity,
  - type of the object $\longrightarrow$ regularization function.

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Inverse problem framework

- Reconstructed image is the solution of:

$$x^{+} = \arg\min_{x \in \mathbb{X}} \underbrace{f_{\text{prior}}(x)}_{\text{regularization}} \quad \text{s.t.} \quad \underbrace{f_{\text{data}}(x|m)}_{\text{interferometry}} \leq \eta_1 \ \text{ and } \ \underbrace{f_{\text{ph}}(x|s)}_{\text{photometry}} \leq \eta_2$$

- with:
  - $f_{\text{prior}}(x)$ regularization;
  - $f_{\text{data}}(x|m)$ “interferometric likelihood”;
  - $f_{\text{ph}}(x|s)$ “photometric likelihood”;
  - η_1 and η_2 tolerance level;
  - $\mathbb{X}$ feasible set *e.g.* $\mathbb{X} = \{x \in \mathbb{R}_{+}^{N}\}$

Interferometric likelihood

- **Sampling**:

$$\boxed{x_{n,\ell} = I(\boldsymbol{\theta}_n, \lambda_\ell)} \quad \text{avec:} \quad \begin{cases} \lambda_\ell & \text{wavelength of } \ell\text{þspectral channel} \\ \boldsymbol{\theta}_n & \text{angular position of pixel } n \\ \boldsymbol{B}_b & \text{projected position of baseline } b \end{cases}$$

- **Direct model**:

$$m_{p,b,\ell} = \sum\nolimits_n H_{p,b,n,\ell}\, x_{n,\ell} + e_{p,b,\ell} \quad \text{in brief:} \quad \boxed{\boldsymbol{m} = \mathbf{H}\cdot\boldsymbol{x} + \boldsymbol{e}}$$

with:

$$H_{p,b,n,\ell} = \begin{cases} +\cos(\boldsymbol{\theta}_n^\top \cdot \boldsymbol{B}_b/\lambda_\ell) & \text{for } p = 1 \text{ (real part)} \\ -\sin(\boldsymbol{\theta}_n^\top \cdot \boldsymbol{B}_b/\lambda_\ell) & \text{for } p = 2 \text{ (imaginary part)} \end{cases}$$

- **Interferometric likelihood term** (assuming Gaussian statistics for the errors)

$$\boxed{f_{\text{data}}(\boldsymbol{x}|\boldsymbol{m}) = \frac{1}{2}(\mathbf{H}\cdot\boldsymbol{x} - \boldsymbol{m})^\top \cdot \mathbf{W} \cdot (\mathbf{H}\cdot\boldsymbol{x} - \boldsymbol{m})} \quad \text{with:} \quad \mathbf{W} = \mathrm{Cov}(\boldsymbol{e})^{-1}$$

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Regularization term: structured sparsity

- GRAVITY object of interest: point-like sources (*e.g.* stellar cluster, galactic center, ...);
  $\Longrightarrow$ our priors are that the object is **spatially sparse** but **not spectrally sparse**
- following Fornasier and Rauhut (2008) and Soulez et al. (2011) we propose to use a structured norm:

$$\boxed{f_{\text{prior}}(\boldsymbol{x}) = \sum_n \left[\sum_\ell x_{n,\ell}^2\right]^{\frac{1}{2}}}$$

$\sqrt{\sum_\ell x_{n,\ell}^2}$ is the ℓ_2 norm of spectrum at nth pixel

- exemple avec 3 pixels × 3 longueurs d'onde:

$$\lambda \uparrow \begin{array}{|c|c|c|} \hline 1 & 0 & 0 \\ \hline 0 & 1 & 0 \\ \hline 0 & 0 & 1 \\ \hline \end{array} \quad \lambda \uparrow \begin{array}{|c|c|c|} \hline 0 & 1 & 0 \\ \hline 0 & 1 & 0 \\ \hline 0 & 0 & 1 \\ \hline \end{array} \quad \lambda \uparrow \begin{array}{|c|c|c|} \hline 0 & 1 & 0 \\ \hline 0 & 1 & 0 \\ \hline 0 & 1 & 0 \\ \hline \end{array}$$

$$\theta \longrightarrow \qquad \theta \longrightarrow \qquad \theta \longrightarrow$$

$$f_{\text{prior}}(\boldsymbol{x}) = 3 \qquad 1 + \sqrt{2} \simeq 2.41 \qquad \sqrt{3} \simeq 1.73$$

- the cost is minimal when the chromatic emission is grouped at the same position;
- **convex** but **non derivable function** regularization function.

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The image reconstruction problem

$$\boldsymbol{x}^{+} = \arg\min_{\boldsymbol{x}\in\mathbb{X}} \underbrace{f_{\text{prior}}(\boldsymbol{x})}_{\text{regularization}} \quad \text{s.t.} \quad \underbrace{f_{\text{data}}(\boldsymbol{x}|\boldsymbol{m})}_{\text{interferometry}} \leq \eta_1 \ \text{ and } \ \underbrace{f_{\text{ph}}(\boldsymbol{x}|\boldsymbol{s})}_{\text{photometry}} \leq \eta_2$$

is convex but

- non linear,
- non derivable,

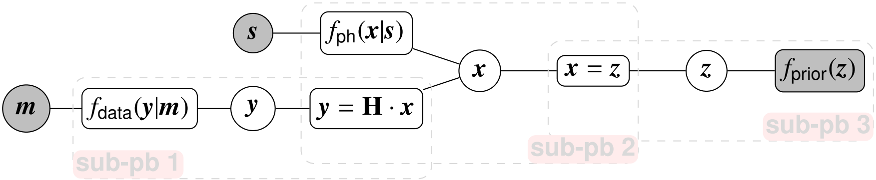
$\longrightarrow$ difficult optimization

Double "splitting"

Problem can be split in three sub-problems using 2 set of auxiliary variables:

$$x^{+} = \arg\min_{x} \mu f_{\text{prior}}(z) + f_{\text{data}}(y|m) + f_{\text{ph}}(x|s) \quad \text{s.t.} \begin{cases} y & = \mathbf{H}\cdot x\,, \\ x & = z\,, \\ z & \geq 0\,. \end{cases}$$

- y: complex visibilities,
- x: 3D intensity distribution,
- z: 3D intensity distribution.



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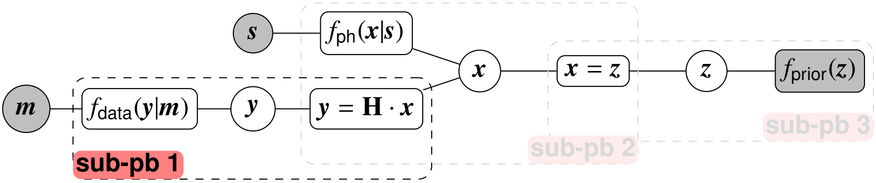

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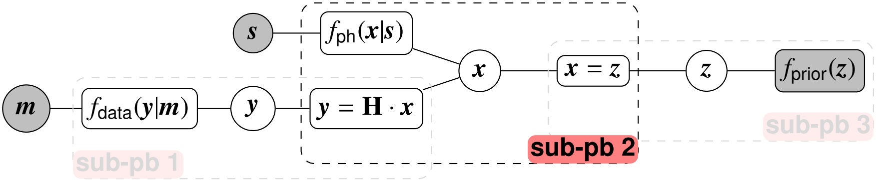

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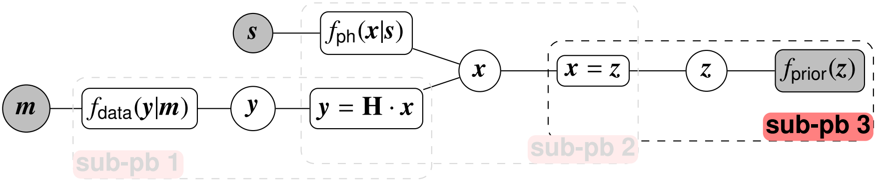

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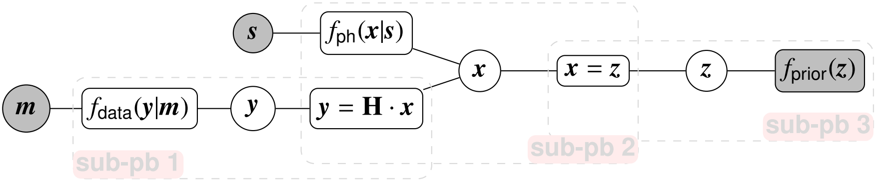

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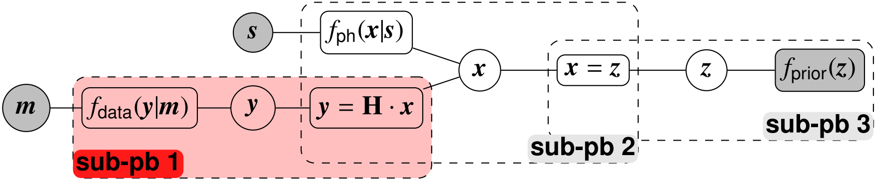
Sub-problem 1

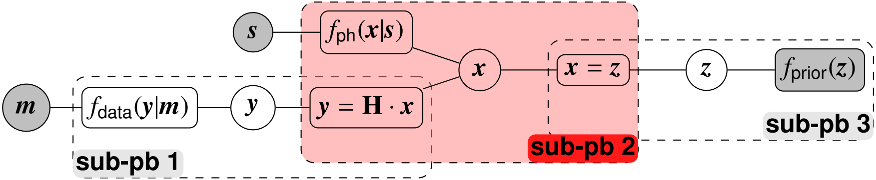


- sub-problem 1 solution:

$$\begin{aligned} \boldsymbol{y}^{+} &= \arg\min_{\boldsymbol{y}} f_{\text{data}}(\boldsymbol{y}|\boldsymbol{m}) + \frac{\rho_1}{2} \left\| \boldsymbol{y} - \widetilde{\boldsymbol{y}} \right\|_2^2 \\ &= \text{prox}_{(1/\rho_1) f_{\text{data}}}(\widetilde{\boldsymbol{y}}) \quad \text{(Moreau proximal mapping operator)} \end{aligned}$$

- $\widetilde{\boldsymbol{y}} = \mathbf{H} \cdot \boldsymbol{x} + \boldsymbol{u}/\rho_1,$
- **separable** on few measurements,
- if $\boldsymbol{m}$ are measured complex visibilities
  $f_{\text{data}}(\boldsymbol{y}|\boldsymbol{m}) = \frac{1}{2}\,(\boldsymbol{y} - \boldsymbol{m})^{\top} \cdot \mathbf{W} \cdot (\boldsymbol{y} - \boldsymbol{m})$
  - **separable** in 2×2 systems (independent complex measurements),
  - analytic solution.

Sub-problem 2

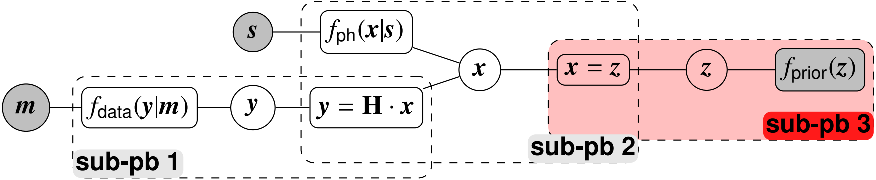


- sub-problem 2 solution:

$$x^{+} = \arg\min_{x} f_{\text{ph}}(x) + \frac{\rho_1}{2} \left\| \mathbf{H} \cdot x - \widetilde{\widetilde{y}} \right\|_2^2 + \frac{\rho_2}{2} \left\| x - \widetilde{x} \right\|_2^2$$

  with $\widetilde{\widetilde{y}} = \mathbf{H} \cdot x - u/\rho_1$ and $\widetilde{x} = z - v/\rho_2$,
- $f_{\text{ph}}(x) = \left\| \sum_k x_k - s \right\|_{W_s}^2$,
- $\mathbf{H} \cdot x$ approximated by non uniform FFT (Keiner et al. 2009),
- x^{+} solution of quadratic problem: **convex**,
- iteratively solved using conjugate gradient.

Sub-problem 3



- sub-problem 3:

$$
\begin{aligned}
z^{+} &= \underset{z \geq 0}{\arg\min}\; \mu f_{\text{prior}}(z) + \frac{\rho_2}{2} \left\| z - \widetilde{z} \right\|_2^2 \\
&= \operatorname{prox}_{(\mu/\rho_2) f_{\text{prior}}}(\widetilde{z})
\end{aligned}
$$

  with $\widetilde{z} = \boldsymbol{x} + \boldsymbol{v}/\rho_2$,
- **separable**
- f_{prior} is **non differentiable**,
- but proximal operator $\operatorname{prox}_{(\mu/\rho_2) f_{\text{prior}}}(\widetilde{z})$ have a closed form solution:

$$
\boxed{z_{n,\ell}^{+} = \begin{cases} (1 - 1/\beta_n)\, \max(0, \widetilde{z}_{n,\ell}) & \text{if } \beta_n > 1 \\ 0 & \text{else} \end{cases}}
$$

with: $\beta_n = (\rho_2/\mu)\, \sqrt{\sum_\ell \max(0, \widetilde{z}_{n,\ell})}$.

ADMM: Alternating Direction of Multipliers Method

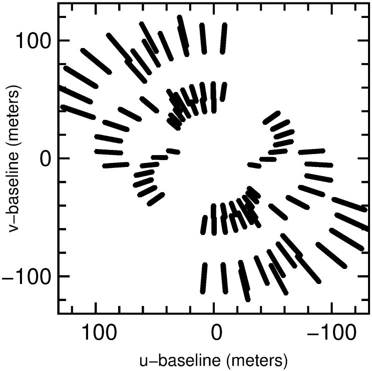
ADMM optimization procedure (Boyd et al. 2010):

- choose μ, ρ_1 et ρ_2,
- initial image$\boldsymbol{x}^{(t=0)}$,
- $\boldsymbol{y}^{(t=0)} = \mathbf{H} \cdot \boldsymbol{x}^{(t=0)}$ and $\boldsymbol{z}^{(t=0)} = \boldsymbol{x}^{(t=0)}$,
- $\boldsymbol{u} = -\partial f_{\text{data}}(\boldsymbol{y})$ and $\boldsymbol{v} = -\partial f_{\text{ph}}(\boldsymbol{x}) + \mathbf{H}^{\top} \cdot \boldsymbol{y}$
- repeat:
  1. $\boldsymbol{z}^{(t+1)} = \text{prox}_{f_{\text{prior}}}(\widetilde{\boldsymbol{z}})$,
  2. solve $\boldsymbol{x}^{(t+1)} = \arg\min f_{\text{ph}}(\boldsymbol{x}^{(t)}) + \frac{\rho_1}{2} \left\| \mathbf{H} \cdot \boldsymbol{x}^{(t)} - \widetilde{\widetilde{\boldsymbol{y}}} \right\|_2^2 + \frac{\rho_2}{2} \left\| \boldsymbol{x}^{(t)} - \widetilde{\boldsymbol{x}} \right\|_2^2$,
  3. $\boldsymbol{y}^{(t+1)} = \text{prox}_{f_{\text{fdata}}}(\widetilde{\boldsymbol{y}})$,
  4. update Lagrange multipliers $\boldsymbol{v}$ and $\boldsymbol{u}$,
  5. $t = t + 1$
- until some convergence.

Simulation

Data simulated by GRAVITY consortium

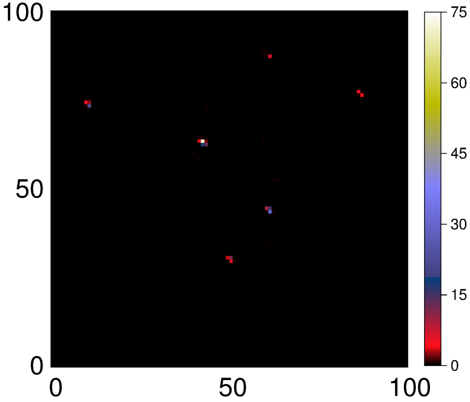
- 6 black bodies from 2000° to 15000°,
- 240 spectral channels from 1.95 µm to 2.45 µm,
- good (u, v) coverage (42 baselines),
- 10080 measurements (complex visibility),
- very good SNR.



(u, v) coverage.

Results

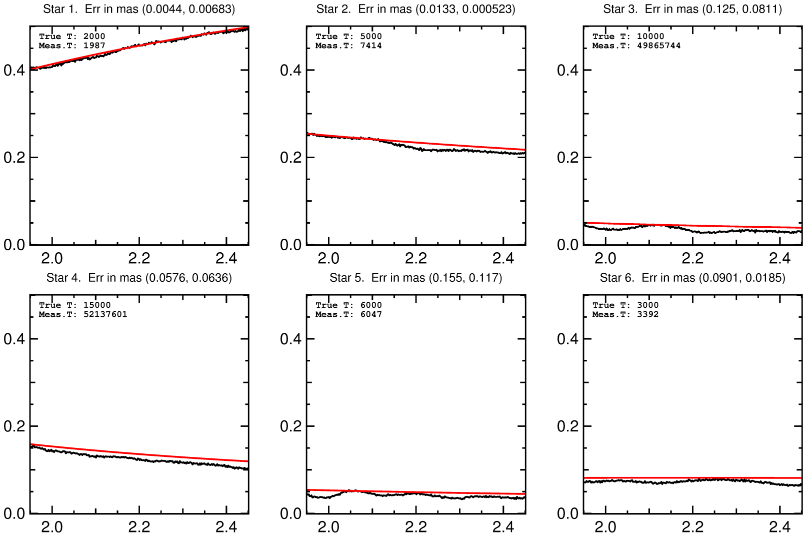
- Sub-problem 1: solving 10080 linear systems of size 2×2,
- Sub-problem 2:
  - $f_{\text{ph}}(\boldsymbol{x}) = ||\mathbf{H} \cdot \boldsymbol{x} - \boldsymbol{s}||^2_{W_s}$ where $\boldsymbol{s} = \mathbf{1}$ (normalization),
  - quadratic problem,
  - solved iteratively.
- Sub-problem 3:
  - proximal operator of $f_{\text{prior}}(\boldsymbol{z}) = \sum_n \sqrt{\sum_\ell z^2_{n,\ell}}$
  - analytic solution,
  - separable: N_{pixels} small problems.



Reconstruction (spectrally integrated, axis are in mas).

Positionning errors ≤ 0.1 **mas** ($\lambda / B_{\text{max}} \sim 4$ mas).

Reconstructed spectra



Normalized spectra (in black) and theoretical spectra (in red) of the 6 stars.

Conclusion

Sources detection in polychromatic interferometry

- a non linear problem solved globally using structured sparsity priors,
- shows the leverage given by spectral dimension in image reconstruction algorithms,

Double “splitting” framework

- One hard problem split in three simpler problems,
- very flexible: boxes can be easily changed,
- changing priors or measurement types $\longrightarrow$ changing one box.

Perspectives

- in the GRAVITY case, compare to greedy methods (CLEAN like),
- use other kind of measurements (VIS2, closures. . .)
- use other *priors*,
- **self-calibration.**

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Bongard, S., Soulez, F., Thiébaut, E., and Pecontal, E. (2011). 3d deconvolution of hyper-spectral astronomical data. *Monthly Notices of the Royal Astronomical Society*, 418(1):258–270.

Boyd, S., Parikh, N., Chu, E., Peleato, B., and Eckstein, J. (2010). Distributed optimization and statistical learning via the alternating direction method of multipliers. *Foundations and Trends in Machine Learning*, 3:1–122.

Fornasier, M. and Rauhut, H. (2008). Recovery algorithms for vector valued data with joint sparsity constraints. *SIAM Journal on Numerical Analysis*, 46(2):577–613.

Keiner, J., Kunis, S., and Potts, D. (2009). Using nfft3 – a software library for various nonequispaced fast fourier transforms. *ACM Trans. Math. Software*, 36:1–30.

Soulez, F., Thiebaut, E., and Denis, L. (2013). Restoration of hyperspectral astronomical data with spectrally varying blur. *EAS Publications Series*, to appear.

Soulez, F., Thiébaut, É., Bongard, S., and Bacon, R. (2011). Restoration of hyperspectral astronomical data from integral field spectrograph. In *3rd*, Lisbon, Portugal.