L'objectif est de résoudre le système d'équation linéaires suivant :

$$\mathbf{A} \cdot \boldsymbol{x} = \boldsymbol{b} \tag{1}$$

avec $\mathbf{A}$ une matrice positive définie.

Gradients conjugués

initialisation:
  compute $\boldsymbol{r}_0 = \boldsymbol{b} - \mathbf{A} \cdot \boldsymbol{x}_0$ for some initial guess $\boldsymbol{x}_0$
  let $k = 0$
until convergence do
  $\rho_k = \boldsymbol{r}_k^{\mathrm{T}} \cdot \boldsymbol{r}_k$
  if $k = 0$, then
    $\boldsymbol{p}_k = \boldsymbol{r}_k$
  else
    $\beta_k = \rho_k / \rho_{k-1}$
    $\boldsymbol{p}_k = \boldsymbol{r}_k + \beta_k \, \boldsymbol{p}_{k-1}$
  endif
  $\boldsymbol{q}_k = \mathbf{A} \cdot \boldsymbol{p}_k$
  $\alpha_k = \rho_k / (\boldsymbol{p}_k^{\mathrm{T}} \cdot \boldsymbol{q}_k)$ (optimal step size)
  $\boldsymbol{x}_{k+1} = \boldsymbol{x}_k + \alpha_k \, \boldsymbol{p}_k$
  $\boldsymbol{r}_{k+1} = \boldsymbol{r}_k - \alpha_k \, \boldsymbol{q}_k$
  $k \leftarrow k + 1$
done

Gradients conjugués préconditionnés

initialisation:
  compute $\boldsymbol{r}_0 = \boldsymbol{b} - \mathbf{A} \cdot \boldsymbol{x}_0$ for some initial guess $\boldsymbol{x}_0$
  let $k = 0$
until convergence do
  solve $\mathbf{M} \cdot \boldsymbol{z}_k = \boldsymbol{r}_k$ for $\boldsymbol{z}_k$ (apply preconditioner)
  $\rho_k = \boldsymbol{r}_k^{\mathrm{T}} \cdot \boldsymbol{z}_k$
  if $k = 0$, then
    $\boldsymbol{p}_k = \boldsymbol{z}_k$
  else
    $\beta_k = \rho_k / \rho_{k-1}$
    $\boldsymbol{p}_k = \boldsymbol{z}_k + \beta_k \, \boldsymbol{p}_{k-1}$
  endif
  $\boldsymbol{q}_k = \mathbf{A} \cdot \boldsymbol{p}_k$
  $\alpha_k = \rho_k / (\boldsymbol{p}_k^{\mathrm{T}} \cdot \boldsymbol{q}_k)$ (optimal step size)
  $\boldsymbol{x}_{k+1} = \boldsymbol{x}_k + \alpha_k \, \boldsymbol{p}_k$
  $\boldsymbol{r}_{k+1} = \boldsymbol{r}_k - \alpha_k \, \boldsymbol{q}_k$
  $k \leftarrow k + 1$
done